Inclusion of Microstrip and Wire Circuits in the FDTD Method

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Abstract — **With the increasing requirement to analyse structures with great complexity and significant size, existing electromagnetic analysis tools are showing their limitations. For instance, an integrated circuit together with its interconnects and package presents a considerable challenge. Methods which can be used include the Finite Difference Time Domain (FDTD) method which is versatile and general but which has impracticable computational requirements for structures with fine detail. In this contribution, an existing thin wire formalism is generalised in order to allow complete microstrip and printed circuits to be modelled in an efficient and rigorous manner.**

Index terms — **FDTD methods, printed circuits, microstrip circuits, wire, PEEC.**

I. INTRODUCTION

The Finite Difference Time Domain (FDTD) method [1]-[2] is a widely used, versatile method for the electromagnetic analysis of large and complex structures. Over recent years many advances have been implemented in order to extend the range of applicability, particularly to small and curved metal structures. Nevertheless, for the analysis of structures such as Printed Circuit Boards (PCBs) and Integrated Circuits (ICs) the fine detail of the geometry presents severe obstacles to the use of the method.

The purpose of this contribution is to introduce a new approach which combines FDTD with a generalisation of the "thin wire formalisms" originally introduced by Holland [3] and further developed by several groups including University of Bristol [4]-[9]. This is aimed at allowing complete circuits to be included within the FDTD method in a rigorous and efficient manner.

II. THE INCLUSION OF WIRES AND STRIPS INTO THE FDTD METHOD

A method of including wire bundles, orientated at arbitrary angles, into the FDTD method is described in detail in [9] and will be used as a starting point for the method described here.

Consider two parallel wires, which may be part of a bundle, passing through the FDTD mesh as shown in Fig. 1. Around each wire, a surface is defined such as the dotted circles shown in the figure [4]. For each surface, a function of space, $w_1(r)$ and $w_2(r)$, is defined which takes the value of unity when r lies on the surface and zero otherwise. Each wire is split into segments and branches, as shown in Fig. 2, with the currents sampled at the centre of each branch and the charge density sampled at the centre of each segment.

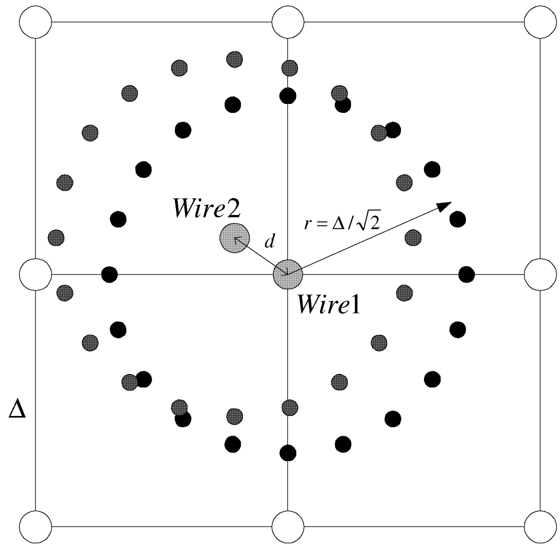

Fig. 1. Two wires showing the shell weighting functions surrounding each

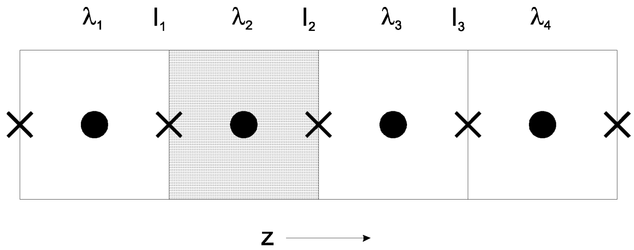

Fig. 2. Discretisation of a wire into segments and branches

Following Holland, the development starts from Faraday's law in cylindrical coordinates

$$\frac{\partial E_z}{\partial r}-\frac{\partial E_r}{\partial z}=\mu\frac{\partial H_\theta}{\partial t} \tag{1}$$

Integration in the radial direction from the surface of wire 1 out to a distance of r gives

$$E_z(r)=\frac{\partial}{\partial z}\left(\int_a^r E_r dr\right)+\mu\frac{\partial}{\partial t}\left(\int_a^r H_\theta dr\right)+E_z(a) \tag{2}$$

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or, equivalently

$$E_z(r) = \frac{\partial}{\partial z}(\phi(r) - \phi(a)) + \frac{\partial}{\partial t}(A_z(r) - A_z(a)) + E_z(a) \tag{3}$$

In [9] and other previous publications, the fields around the wires are assumed to be those of an infinite bundle of parallel wires in which case the potentials can be written as follows

$$A_z = \frac{\mu}{2\pi} \sum_i I_i \ln(r_i) \tag{4}$$

$$\phi = \frac{1}{2\pi\varepsilon} \sum_i \lambda_i \ln(r_i) \tag{5}$$

where r_i is the radial distance from the centre of the wire to the observation point. λ and I are the line charge densities and currents respectively.

In order to analyse wire or microstrip circuits, or wires which are not parallel, the potentials must be calculated by integration over the wire branches and segments.

The update equation for the wire currents is now written as

$$\mathbf{I}^{n+1} = \left(\mathbf{R}^{+}\right)^{-1}\left(\mathbf{R}^{-}\mathbf{I}^{n} - c^2 \delta t \mathbf{D} \boldsymbol{\lambda} + \delta t \mathbf{L}^{-1}\mathbf{X}\right) \tag{6}$$

where $\boldsymbol{I}$ is the branch current vector and the superscript indicates the time iteration number. $\boldsymbol{\lambda}$ is the segment charge density vector. The matrix, $\boldsymbol{R}$, accounts for finite resistivity of the wire and is the unit matrix for perfect conductors. The matrix, $\boldsymbol{D}$, implements the central difference approximation to the derivative. For a single wire it has the following structure

$$\mathbf{D} = \frac{1}{\Delta}\mathbf{C} = \frac{1}{\Delta}\begin{pmatrix} 1 & -1 & 0 & \vdots & 0 \\ 0 & 1 & -1 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{pmatrix} \tag{7}$$

where Δ, is the wire segment length. In circuit analysis terms, the matrix, $\mathbf{C}$, can be interpreted as the connection matrix which defines the segments to which each branch is connected.

The inductance matrix, L, contains the "in-cell" self and mutual inductances between segments as described in [9] based on the "shell average" used in [4] and [9]

$$L_{ij} = \left\langle A_j, w_i \right\rangle - A_j\left(r_{ij}\right) \tag{8}$$

and

$$X_i = \left\langle E, w_i \right\rangle + V_{si} / \Delta \tag{9}$$

where A_j is the magnetic vector potential resulting from unit current flowing in branch j, r_{ij} is the distance between the i^{th} and j^{th} wires and the angle brackets denote integration over all space.

$\langle E, w_i \rangle$ is the average longitudinal E field on the i^{th} surface and V_{si} is any voltage source which exists across segment, i.

The update equation for the charge density on the wire is derived from the continuity equation as:

$$\boldsymbol{\lambda}^{n+1/2} = \boldsymbol{\lambda}^{n-1/2} - \Delta t \mathbf{D}^T \mathbf{I}^n \tag{10}$$

In the formulations of [8] and [9], as well as the field distribution around the wires being assumed to be the same as if the wire was infinitely long, the mutual inductances are assumed to be non-zero only between adjacent branches. For example, referring to Fig. 3, the mutual inductance between branches 1 and 11 are calculated as that between two infinitely long parallel wires whereas the mutual inductance between branch 2 and branch 11 are taken as zero. For the structures examined in [8] and [9] the approximation does not lead to significant inaccuracy. However for dealing with wires which are not parallel and with complete discretised circuits, a more rigorous approach is necessary.

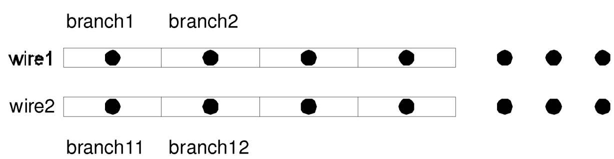

Fig. 3. Two discretised parallel wires

III. Extension to Microstrip Circuits

As an example, consider part of a microstrip circuit as shown in Fig. 4. As with the wire, this structure is split into segments and branches but in this case the discretisation is two dimensional.

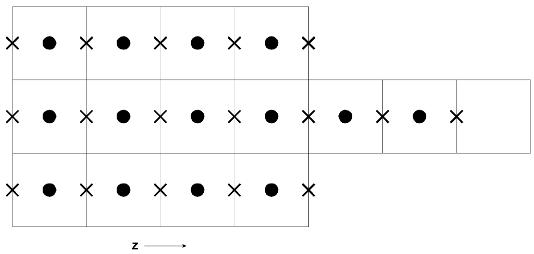

Fig. 4. Part of a microstrip circuit containing a step discontinuity

If the segments and branches are numbered then a connection matrix can be formed and the update equation written as

$$\mathbf{I}^{n+1} = \left(\mathbf{R}^{+}\right)^{-1}\left(\mathbf{R}^{-}\mathbf{I}^{n-1} + \delta t\mathbf{L}^{-1}\left(\mathbf{X} - \mathbf{CP}\boldsymbol{\lambda}^{n+1/2}\right)\right) \tag{11}$$

$$\boldsymbol{\lambda}^{n+1/2} = \boldsymbol{\lambda}^{n-1/2} + \delta t\mathbf{C}^{T}\mathbf{I}^{n} \tag{12}$$

where the matrix, $\boldsymbol{P}$, is the inverse capacitance matrix for the structure.

Comparing equations (11) and (12) with (6) and (10) shows that the latter is a generalisation of the former in that the connection matrix for the circuit is used in place of the central difference matrix and that the capacitance matrix is explicitly included. In addition, in the existing multiwire formalisms, the inductance values are calculated under the assumption that the wires are infinitely long. This approximation is not adequate when dealing with circuits and all the mutual inductances must be explicitly calculated. This can be done in the following way.

If it is assumed that the current is constant on each branch then the magnetic vector potential at point (x,y,z) caused by a segment centred at the origin, is given by [10]

$$A_{zj}(x,y,z) = \frac{\mu}{2\pi b_j}\begin{pmatrix} I\left(x-a_j, y-b_j, z\right) \\ +I\left(x+a_j, y+b_j, z\right) \\ -I\left(x-a_j, y+b_j, z\right) \\ -I\left(x+a_j, y-b_j, z\right) \end{pmatrix} \tag{13}$$

where

$$\begin{aligned} I(x,y,z) &= \int\int \frac{dxdy}{\sqrt{x^2+y^2+z^2}} \\ &= x\ln\left(y+\sqrt{x^2+y^2+z^2}\right) \\ &\quad + y\ln\left(x+\sqrt{x^2+y^2+z^2}\right) \\ &\quad + 2z\tan^{-1}\left(\frac{x+y+\sqrt{x^2+y^2+z^2}}{z}\right) \end{aligned} \tag{14}$$

a_j and b_j are the half width and length of the j^{th} branch respectively. A similar expression can be derived for the electric scalar potential which can then be used to calculate the mutual capacitances.

IV. Comparison with PEEC Methods

There are similarities between the method outlined above and the PEEC based methods, such as [10], however there are several important differences as follows.

The inductance matrices used in [10] are the partial inductances between each pair of branches and are calculated by integrating the magnetic vector potential caused by unit current in the source patch over the area of the victim patch. In this approach, the "in-cell" inductances are used which involves integrating the magnetic vector potential caused by unit current flowing in the source branch over a surface surrounding the victim patch analogously with equation (8). The reason that the inductances are not the same in the two methods can be traced to the fact that some of the coupling is accounted for by the FDTD mesh rather than it all being accounted for by the mutual inductances.

A comparison of these two different inductances for two wire branches of unit length and with various separations in the longitudinal and transverse directions can be seen in Fig. 5 and Fig. 6. Here, the mutual inductance between two branches is plotted as a function of transverse distance between them for four different longitudinal separations. It can be seen from Fig. 6 that there is significant mutual coupling even when the separation is large. In contrast to this, Fig. 5 shows that, for the same separation, the mutual coupling drops to low values even when the separation is small. In fact, it can be shown for the case of wires, that if the transverse separation between source and victim branches is such that the source branch is outside the integration surface surrounding the victim branch then the mutual coupling tends to zero.

This has important consequences. Not only does this reduce the number of interactions which need to be included, thus reducing computational requirements, but the necessity of including retardation effects with its accompanying complications, [11], and problems of numerical instability, [12], is avoided. Effectively, all the long range interactions are accounted for by the FDTD algorithm and the short range, detailed interactions are dealt with by the in-cell inductances and capacitances.

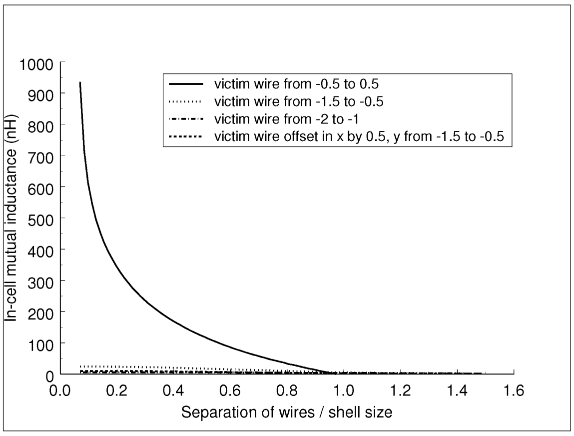

Fig. 5. Mutual "in-cell inductance" between two wire segments in the hybrid case

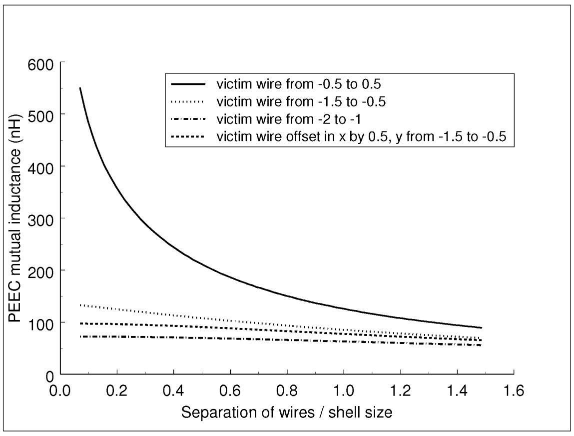

Fig. 6. Mutual inductance between two wire segments in the PEEC case

V. Conclusion

In this contribution a method has been described which will allow complete microstrip or wire circuits to be included within the FDTD method but without the need for a very fine mesh. This approach, which is based on an extension of the "thin wire formalisms" previously published, paves the way for the practical analyses of large complex Printed Circuit Boards and Integrated Circuits together with their environments.

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